Ecuaciones Diferenciales Ordinarias (EDO)

Son ecuaciones con derivadas.

Ec de primer orden (derivadas primeras) **por el método de separación de variables:**

- *Si son de la forma:* $f(y)\cdot y' = f(x)$ → pasar a la forma: $f(y)dy = g(x)dx$

Y luego integrar a ambos lados: $\int f(y)\,dy = \int g(x)\,dx + c$

Ejemplo 1: $\boldsymbol{y' + y\cdot senx = 0}$ con las condiciones iniciales: $y(0) = e$

1º Pasar a la forma: $\frac{dy}{dx} + y\cdot senx = 0$

2º Separar variables: $\frac{dy}{dx} = -y\cdot senx \rightarrow \frac{dy}{y} = senx\,dx$

3º Integrar ambos miembros: $\int \frac{1}{y}\,dy = \int senx\,dx \rightarrow Ln\,y = cos\,x + C$

4º Despejar *y*: $y = e^{\cos x + C} = e^{\cos x}\cdot e^{C} = k\cdot e^{\cos x}$

5º Aplicar las cond iniciales: $k\cdot e^{\cos 0} = e \rightarrow k = 1$ ó $C = 0$

Ejemplo 2: $\boldsymbol{y' = e^{2x+3y}}$ con las condiciones iniciales: $y(0) = 0$

1º Pasar a la forma: $\frac{dy}{dx} = e^{2x}\cdot e^{3y}$

2º Separar variables: $\frac{dy}{e^{3y}} = e^{2x}dx$

3º Integrar ambos miembros: $\int \frac{dy}{e^{3y}} = \int e^{2x}dx \rightarrow \int e^{-3y}dy = \frac{1}{2}\int e^{2x}\cdot 2\,dx \rightarrow -\frac{1}{3}e^{-3y} = \frac{1}{2}e^{2x} + C$

4º Despejar *y*: $e^{-3y} = -\frac{3}{2}e^{2x} + K \rightarrow -3y = \ln\left(-\frac{3}{2}e^{2x}\right) \rightarrow y = -\frac{1}{3}\ln\left(-\frac{3}{2}e^{2x}\right)$

5º Aplicar las cond. iniciales: $e^{-3\cdot 0} = -\frac{3}{2}e^{0} + K \rightarrow K = \frac{5}{2} \rightarrow y = -\frac{1}{3}\ln\left(-\frac{3}{2}e^{2x} + 5/2\right)$

- *Si son de la forma* $y' = f(ax+by)$ → cambiar variable: $z = ax + by$

Ejemplo: $\boldsymbol{y' - e^{x}e^{y} = -1}$

$y' = e^{x+y} - 1$ cambiar variable: $z = x + y \rightarrow \frac{dz}{dx} = 1 + \frac{dy}{dx} \rightarrow y' = z' - 1$

$z' - 1 = e^{z} - 1 \rightarrow z' = e^{z} \rightarrow \frac{dz}{dx} = e^{z} \rightarrow \frac{dz}{e^{z}} = dx \rightarrow \int e^{-z}dz = \int dx$

$x = -e^{-z} + \mathrm{C} \rightarrow ln\,|c - x| = -z \rightarrow$ descambiamos: $ln\,|c - x| = -x - y$

$y = -x - ln\,|c - x|$

- *Si son de la forma* $y' = f(y/x)$ → cambiar variable: $y = u\cdot x \rightarrow y' = u'\cdot x + u\cdot x' = u'\cdot x + u$

Ejemplo: $\boldsymbol{y' = \frac{2xy - y^2}{x^2}}$

1º Pasar a la forma: $y' = \dfrac{\frac{2xy}{x^2} - \frac{y^2}{x^2}}{\frac{x^2}{x^2}} = 2\left(\frac{y}{x}\right) - \left(\frac{y}{x}\right)^2$

2º Cambiar variable $u = \frac{y}{x}$: $y' = 2(u) - (u)^2 \rightarrow u'\cdot x + u = 2u - u^2$

3º Separar variables: $\frac{du}{dx}\cdot x = u - u^2 \rightarrow \frac{du}{u-u^2} = \frac{dx}{x}$

3º Integrar ambos miembros: $\int \frac{du}{u(1-u)} = \int \frac{dx}{x} \rightarrow \int \frac{A}{u}du + \int \frac{B}{1-u}du = \ln x + \ln C$ $(u \neq 0\ ;\ u \neq 1)$

$\ln u - \ln(1-u) = \ln x + \ln C \rightarrow \cancel{\ln}\left(\frac{u}{1-u}\right) = \cancel{\ln}(kx) \rightarrow \frac{u}{1-u} = kx$

4º Descambiamos $u = \frac{y}{x}$: $\dfrac{\frac{y}{x}}{1-\frac{y}{x}} = kx \rightarrow \frac{y}{x-y} = kx \rightarrow y = \frac{kx^2}{1+kx}$ (solución general de la ec. diferencial)

Para u=0: $y = 0\cdot x = 0$; Para u=1: $y = 1\cdot x = x$ (soluc. Particulares)

• *Si son de la forma* $y' = f\left(\frac{ax+by+c}{a_1x+b_1y+c_1}\right)$ es la división de 2 rectas que,

→ si son paralelas, hacer cambio de variable: $z = ax + by$,

→ si se cortan, hacer el cambio: $X = x - x_o$; $Y = y - y_o$ (homogénea)

Ejemplo 1: $\boldsymbol{y' = \frac{-2x+4y-6}{x+y-3}}$ como sus coef. No son proporcionales, se cortan (sistema) en el punto P(1,2)

Cambiar variable: $X = x - 1$; $Y = y - 2$ y sus derivadas: $X' = 1$; $Y' = y'$ → $x = X + 1$; $y = Y + 2$

$y' = \frac{-2(x+1)+4(y+2)y-6}{x+1+y+2-3}$ → $y' = \frac{-2x+4}{x+y}$ como es del tipo: $y' = f(y/x)$ $cambiamos$: $u = \frac{y}{x}$

$y' = \frac{-2+4\frac{y}{x}}{1+\frac{y}{x}}$ → $u' \cdot x + u = \frac{-2+4u}{1+u}$ → $u' \cdot x = \frac{-2+4u}{1+u} - \frac{u}{1} = \frac{-2+4u-u-u^2}{1+u} = \frac{-u^2+3u-2}{1+u}$

$\frac{du}{dx} \cdot x = \frac{-u^2+3u-2}{1+u}$ → $\frac{dx}{x} = \frac{1+u}{-u^2+3u-2} du$ → $\int \frac{dx}{x} = \int \frac{1+u}{-u^2+3u-2} du$ → $ln\, x + C = \int \frac{A}{u-1} du + \int \frac{B}{u-2} du$

(A=1 ; B=-3) siempre que u≠1 ; u≠2

$ln\, x + lnC = 2\ln(u-1) - 3\ln(u-2) = ln\frac{(u-1)^2}{(u-2)^3}$ → $\cancel{ln}(kx) = \cancel{ln}\frac{(u-1)^2}{(u-2)^3}$

$Kx = \frac{(u-1)^2}{(u-2)^3}$ descambiamos: $Kx = \frac{(y/x-1)^2}{(y/x-2)^3}$ → $K = \frac{(y-x)^2}{(y-2x)^3}$ → $(y-x)^2 = K(y-2x)^3$

descambiamos (X,Y): $(Y - X)^2 = K(Y - 2X)^3$ → $(y - 2 - (x-1))^2 = K(y-2x)^3$

$(y - x - 1)^2 = K(y-2x)^3$ → soluc. general ($K \in \mathbb{R}$)

$Si\, u = 1$: $1 = \frac{Y}{X}$ → $Y = X$ → $y - 2 = x - 1$ → $y = x + 1$ soluc. particular 1

$Si\, u = 2$: $2 = \frac{Y}{X}$ → $2Y = X$ → $y - 2 = 2(x-1)$ → $y = 2x$ soluc. particular 1

Ejemplo 2: $\boldsymbol{y' = \frac{x-y+1}{x-y-2}}$ como sus coef. son proporcionales, son rectas paralelas.

Cambiamos variable: $z = ax + by = 1x - 1y$ → $z' = 1 - y'$ → $y' = 1 - z'$

Separamos: $1 - z' = \frac{z-1}{z-2}$ → $-z' = \frac{z-1}{z-2} - \frac{1}{1} = \frac{1}{2-z}$ → $\frac{dz}{dx} = \frac{1}{z-2}$ → $(z-2)dz = dx$

Integramos: $\int (z-2)dz = \int dx$ → $\frac{(z-2)^2}{2} = x + C$ → $(z-2)^2 = 2x + 2C = 2x + K$

Descambiamos: $(x - y - 2)^2 = 2x + K$

Ejercicios:	*Soluciones:*
1. $\frac{dy}{dx} = \sin 5x$	$dy = sen\, 5x \cdot dx \rightarrow \int dy = \int sen\, 5x\, dx \rightarrow y = -\frac{1}{5} cos\, 5c + C$
2. $\frac{dy}{dx} = (x+1)^2$	$dy = (x+1)^2\, dx \rightarrow \int dy = \int (x+1)^2\, dx \rightarrow y = \frac{1}{3}(x+1)^3 + C$
3. $dx + e^{3\dot{x}}\, dy = 0$	$e^{3x}\, dy = -\, dx \rightarrow dy = -e^{-3x}\, dx \rightarrow \int dy = -\int e^{-3x}\, dx \rightarrow$ $y = -\left(-\frac{1}{3}e^{-3x} + C\right) \rightarrow y = \frac{1}{3}e^{-3x} + C$
4. $dx - x^2\, dy = 0$	$x^2\, dy = dx \rightarrow dy = x^{-2}\, dx \rightarrow \int dy = \int x^{-2}\, dx \rightarrow y = -x^{-1} + C = -\frac{1}{x} + C$
5. $(x+1) \cdot \frac{dy}{dx} = x + 6$	$dy = \frac{x+6}{x+1} dx \rightarrow \int dy = \int \frac{x+6}{x+1} d\dot{x} \rightarrow \int \left(1 + \frac{5}{x+1}\right) dx = x + 5\ln\lvert x+1\rvert + C$
6. $e^x \frac{dy}{dx} = 2x$	$dy = 2x \cdot e_{dx}^{-x} \rightarrow \int dy = \int 2x \cdot e^{-x}\, dx = -2x \cdot e^{-x} - 2 \cdot e^{-x} + C$
7. $x \cdot y' = 4y$	$x \cdot \frac{dy}{dx} = 4y \rightarrow y^{-1} \cdot dy = 4x^{-1}\, dx \rightarrow \ln y = 4\ln x + 4\ln C \rightarrow y = C^4 \cdot x^4$
8. $\frac{dy}{dx} + 2xy = 0$	$\frac{dy}{dx} = -2xy \rightarrow y^{-1}\, dy = -2 \times dx \rightarrow \int y^{-1}\, dy = \int -2x\, dx \rightarrow \ln y = -x^2 + C$ $y = e^{-x^2} \overset{+C}{\rightarrow} y = C \cdot e^{-x^2}$
9. $\frac{dy}{dx} = \frac{y^3}{x^2}$	$y^{-3}\, dy = x^{-2}\, d\dot{x} \rightarrow \int y^{-3}\, dy = \int x^{-2}\, dx \rightarrow -\frac{1}{2}y^{-2} = -x^{-1} + C \rightarrow$ $y^{-2} = 2x^{-1} - 2C \rightarrow \dot{y}^{-2} = 2x^{-1} + C \rightarrow y = \sqrt{\frac{x}{2} + c}$
10. $\frac{dy}{dx} = \frac{y+1}{x}$	$\frac{1}{y+1} dy = \frac{1}{x} dx \rightarrow \int \frac{1}{y+1} dy = \int \frac{1}{x} dx \rightarrow \ln\lvert y+1\rvert = \ln 1 \times \lvert + \ln 1\, c\rvert = \ln\lvert cx\rvert$ $\rightarrow y + .1 = cx \rightarrow y = cx - 1$

Ejercicios ampliación:

11. $\frac{dx}{dy} = \frac{x^2 \cdot y^2}{2+x^2}$

12. $\frac{dx}{dy} = \frac{1+2y^2}{y \cdot Senx}$

13. $\frac{dy}{dx} = e^{3x+2y}$

14. $e^x \cdot y \cdot \frac{dy}{dx} = e^{-y} + e^{-2x-y}$

15. $(4y + yx^2)\, dy - (2x + xy^2)\, dx = 0$

16. $(1 + x^2 + y^2 + x^2y^2)\, dy = y^2\, dx$

17. $2y(x+1)\, dy = x\, dx$

18. $x^2y^2\, dy = (y+1)\, dx$

19. $y \ln x \frac{dx}{dy} = \left(\frac{y+1}{x}\right)^2$

20. $\frac{dy}{dx} = \left(\frac{2y+3}{4x+5}\right)^2$

21. $\frac{ds}{dr} = ks$

22. $\frac{dQ}{dt} = k(Q - 70)$

23. $\frac{dN}{dt} + N = Nte^{t+2}$

24. $sec^2\, x\, d^{y\cdot} + csc\, y\, dy = 0$

25. $sen\, 3x\, dx + 2y\, cos^3 3x\, dy$

26. $sen\, 3x\, dx + 2y \cos^3 3x\, dy = 0$

27. $e^y\, sec\, 2x\, dx + cos\, x \cdot (e^{2y} - y)\, dy = 0$

28. $\sec x\, dy = x \cdot cotg\, y\, dx$

29. $(e^y + 1)^2 e^{-y}\, dx + (e^x + 1)^3\, dy = 0$

30. $\frac{y\, dy}{x\, dx} = (1 + x^2)^{-\frac{1}{2}}(1 + y^2)^{1/2}$

31. $(y - yx^2)\frac{dy}{dx} = (y+1)^2$

32. $2\frac{dy}{dx} = \frac{xy+3x-y-3}{xy-2x+4y=8}$

Soluciones:

11. $\frac{dx}{dy} = \frac{x^2 \cdot y^2}{2+x^2}$

$¿\frac{1+x}{x^2} dx = y^2\, dy \to y^2\, dy = \frac{1+x}{x^2} dx \to \int y^2\, dy = \int \frac{1+x}{x^2} dx \to$
$\int y^2\, dy = \int \frac{1}{x2} dx + \int_x^{\perp} dx = -x^{-1} + \ln x + C \to \frac{1}{3} y^3 = -x^{-1} + \ln \dot{x} + c \to$
$y^3 = -3x^{-1} + 3\ln x + c$

12. $\frac{dx}{dy} = \frac{1+2y^2}{y \cdot Senx}$

$\operatorname{sen} x\, dx = \frac{1+2y^2}{x} dy \to (y^{-1} + 2y) dy = \operatorname{sen} x\, dx \to$
$\int (y^{-1} + 2y)\, dy = \int sen\ x\ dx \to \ln y + y^2 = -\cos x + C$

13. $\frac{dy}{dx} = e^{3x+2y}$

$\frac{dy}{dx} = e^{3x} \cdot e^{2y} \to e^{-2y}\, dy = e^{3x}\, dx \to \int e^{-2y}\, dy = \int \dot{e}^{3x}\, dx \to$
$-\frac{1}{2} e^{-2y} = \frac{1}{3} e^{3x} + c \to \to -3e^{-2y} = 2e^{3x} + c$

14. $e^x \cdot y \cdot \frac{dy}{dx} = e^{-y} + e^{-2x-y}$

Sol: $e^x \cdot y \frac{dy}{d\dot{x}} = e^{-y}\left(1 + e^{-2\dot{x}}\right) \to ye^y\, dy = e^{-x}(1 + e^{-2x}) dx \to\ y \cdot e^y\, dy = (e^{-x} + e^{-3x})\, dx \ \to\ \int ye^y\, dy = \int (e^{-x} + e^{-3x})\, dx \to y \cdot e^y - e^y = -e^{-x} - \frac{1}{3} e^{-3x} + C$

15. $(4y + yx^2)\, dy - (2x + xy^2)\, dx = 0$

Sol: $y(4 + x^2)\, dy = x(2 + y^2)\, dx \ \to\ \frac{Y}{2+y^2} dy = \frac{X}{4+X^2} dx \to \frac{1}{2} \int \frac{2y}{2+y^2} dy = \frac{1}{2} \int \frac{2x}{4+x^2} dx \to$

$\frac{1}{2} \ln(2 + y^2) = \frac{1}{2} \ln(4 + x^2) + \frac{1}{2} \ln c \ \to\ \ln(2 + y^2) = \ln C \cdot (4 + x^2) \ \to$

$2 + y^2 = C(4 + x^2) \to y^2 = C(4 + x^2) - 2$

16. $(1 + x^2 + y^2 + x^2 y^2)\, dy = y^2\, dx$

Sol: $[1 + x^2 + y^2(1 + x^2)]\, dy = y^2\, dx \ \to\ [(1 + x^2) + y^2(1 + x^2)]\, dy = y^2\, dx \to$

$(1 + x^2)(1 + y^2)\, dy = y^2\, dx \to \frac{1+y^2}{y^2} dy = \frac{1}{1+x^2} dx \to \int \frac{1+y^2}{y^2} dy = \int \frac{1}{1+x^2} dx \to$

$\int (y^{-2} + 1)\, dy = \int \frac{1}{1+x^2} dx \to -y^{-1} + y = \arctan(x) + C \to y - \frac{1}{y} = \arctan x + C$

17. $2y(x + 1)\, dy = x\, dx$

Sol: $2y\, dy = \frac{x}{x+1} dx \to \int 2y\, dy = \int \frac{x}{x+1} dx \ \to y^2 = x - \ln|x + 1| + C$

18. $x^2 y^2\, dy = (y + 1)\, dx$

Sol: $\frac{y^2}{y+1} dy = \frac{1}{x \cdot 2} dx \to \int \frac{y^2}{y+1} dy = \int x^{-2}\, dx \ \to \int \left(y - 1 + \frac{1}{y+1}\right) dy = \int x^{-1}\, dx \to$

$\frac{1}{2} y^2 - y + \ln|y + 1| = -\frac{1}{x} + C$

19. $y \ln x \frac{dx}{dy} = \left(\frac{y+1}{x}\right)^2$

Sol: $y \ln y = \frac{(y+1)^2}{x^2} \to x^2 \ln x\, dx = \frac{(y+1)^2}{y} dy \to \int \frac{(y+1)^2}{y} dy = \int x^2 \cdot ln\, x\, dx \to$

$\int \frac{y^2+2y+1}{y} dy = \int x^2 \cdot ln\, x\, dx \to \int (\dot{y} + 2 + y^{-1})\, dy = \int x^2\, ln\, x\, dx \to \frac{1}{2} y^2 + 2y + \ln|y| = \frac{1}{3} x^3 \ln x - \frac{1}{9} x^3 + C$

20. $\frac{dy}{dx}=\left(\frac{2y+3}{4x+5}\right)^2$

Sol: $\int \frac{dy}{(2y+3)^2}=\int \frac{dx}{(4x+5)^2} \to -\frac{1}{2}(2y+3)^{-1}=-\frac{1}{4}(4x+5)^{-1}-\frac{c}{4} \to 2(y+3)^{-1}=(4x+5)^{-1}+c$

$y=4x+1+C$

21. $\frac{ds}{dr}=ks$

Sol: $\frac{dS}{S}=k\,dr \to \int \frac{dS}{S}=\int k\,dr \to ln|S|=kr+c_1 \to S=e^{kr+c} \to S=e^c\cdot e^{kr} \to s=ce^{kr}$

22. $\frac{dQ}{dt}=k(Q-70)$

Sol: $\frac{dQ}{Q-70}=k\,dt \to \int \frac{dQ}{Q-70}=\int k\,dt \to ln(Q-70)=kt+c \to Q=ce^{kt}+70$

23. $\frac{dN}{dt}+N=Nte^{t+2}$

Sol: $\frac{dN}{dt}=Ne^{t+2}\cdot N \to \frac{dN}{dt}=N(te^{t+2}-1)dt \to \int \frac{dN}{N}=\int\left(te^{t+2}-1\right)dt \to ln\,N=\int te^{t+2}dt-t+c$

Por partes: $\int t\cdot e^{t+2}\,dt=te^{t+2}-\int e^{t+2}\,dt=te^{t+2}-e^{t+2} \to ln\,N=te^{t+2}-e^{t+2}-t+c$

24. $sec^2\,x\,d^{y\cdot}+csc\,y\,dy=0$

Sol: $sec^2\,x\,d^{y\cdot}=-csc\,y\,dy \to \frac{dy}{csc\,y}=-\frac{dx}{sec^2\,x} \to sin\,y\,dy=-cos^2\,x\,dx \to$

$\int seny\,dy=-\int cos^2\,x\,dx \to -\cos y=-\int cos^2\,x\,dx \to \cos y=\int cos^2\,x\,dx \to cos\,y=\frac{cos\,x\cdot senx+x}{2}+C$

$\to 2cos\,y=cos\,x\cdot senx+x+C \to 4cosy=2senx\cdot cosx+2x+c \to 4cosy=sen2x+2x+c$

25. $sen\,3x\,dx+2y\,cos^3 3x\,dy$

Sol: $2y\,cos^3 3x\,dy=-sen\,3x\,dx \to 2y\,dy=\frac{-sen\,3x}{cos^3 3x}dx$

$\to \int 2y\,dy=\int\frac{-sen\,3x}{cos^3 3x}dx \to y^2=\int\frac{-sen\,3x}{cos^3 3x}dx \to y^2=\frac{1}{3}\int u^{-3}\,du\ ;\ \{u=cos3x\ ;\ \frac{1}{3}du=-sen3x\}$

$\to y^2=\frac{1}{6}\,u^{-2}+c \to y^2=-\frac{1}{6}(\cos 3x)^{-2}+C \to y^2=-\frac{1}{6}\sec^2 3x+C \to y=\pm\sqrt{c-\frac{1}{6}sec^2\,3x}$

26. $sen\,3x\,dx+2y\cos^3 3x\,dy=0$

Sol: $2y\,cos^3\,3x\,dy=-sin\,3x\,dx \to 2y\partial y=-\frac{\text{sen}\,3x}{\cos^3 3x}dx \to \int 2y\,dy=\int\frac{-\text{sen}\,3x}{\cos^3 3x}dx \to y^2=\frac{1}{3}\int u^{-3}\,du$

$\to y^2=-\frac{1}{6}(cos\,3x)^{-2}+C \to y^z=-\frac{1}{6}sec^2(3x)+C \to y=\pm\sqrt{c-\frac{1}{6}\sec^2(3x)}$

27. $e^y\,sec\,2x\,dx+cos\,x\cdot(e^{2y}-y)\,dy=0$

Sol: $\to e^{-y}(e^{2y}-y)\,dy=\frac{-sen\,2x}{cos\,x} \to (e^y-ye^{-y})\,dy=-\frac{2\sin x\cos x}{\cos x}dx \to (e^y-ye^{-y})\,dy=$
$-2senx\,dx \to \int(e^y-ye^{-y})\,dy=-\int 2\,sin\,x\,dx \to e^y-(-e^{-y}-ye^{-y})=2\,cos\,X+c \to$
$e^y+e^{-y}+ye^{-y}=2cosx+c$

28. $\sec x\,dy=x\cdot cotg\,y\,dx$

Sol: $tany\,dy=x\,cosx\,dx \to \int\tan dy=x\cdot\cos x\,dx \to \ln(\sec y)=\cos x+x\sin x$

$\sec y=e\overset{\cos x+x\sin x}{\to}\ y=\arcsin\left(e^{\cos x+x\sin x}\right)$

29. $(e^y+1)^2e^{-y}\,dx+(e^x+1)^3\,dy=0$

Sol: $\frac{e^y}{(e^y+1)^2}dy=-\frac{e^x}{(e^x+1)^3}dx \to \int\frac{e^y}{(e^y+1)^2}dy=-\int\frac{e^x}{(e^x+1)^3}dx \to -(e^y+1)^{-1}=\frac{1}{2}(e^x+1)^{-2}+\frac{1}{2}C$

$\to \frac{1}{2}(e^x+1)^{-2}-(e^y+1)^{-1}=\frac{1}{2} \to (e^x+1)^{-2}-2(e^y+l)^{-1}=C$

30. $\frac{y\,dy}{x\,dx} = (1+x^2)^{-\frac{1}{2}}(1+y^2)^{1/2}$

Sol: $\frac{y}{(1+y^2)^{\frac{1}{2}}}dy = \frac{x}{(1+x^2)^{\frac{1}{2}}}dx \to \int \frac{y}{(1+y^2)^{\frac{1}{2}}}dy = \int \frac{x}{(1+x^2)^{\frac{1}{2}}}dx \to \int (1+y^2)^{-1/2}\, 2y\,dy =$
$\int (1+x^2)^{-1/2}\, 2x\,dx \to 2(1+y^2)^{1/2} = 2(1+x^2)^{1/2} + 2c \to (q+y^2)^{1/2} = (1+x^2)^{1/2} + c$
$(1+y^2)^{1/2} - (1+x^2)^{1/2} = c$

31. $(y - yx^2)\frac{dy}{dx} = (y+1)^2$

Sol: $y(1-x^2)\,dy = (y+1)^2\,dx \to \frac{y}{(y+1)^2}dy = \frac{1}{(1-x^2)}dx \to \int \frac{y}{(y+1)^2}dy = \int \frac{1}{(1+x^2)}dx \to$
$u = y+1 \to du = dy \to y = u-1$; $\int \frac{y}{(y+1)^2}dy = \int \frac{u-1}{u^2}du = \int \frac{1}{u}du - \int u^2\,du = \ln|y+1| +$
$(y+1)^{-1} + C$; $\int \frac{1}{(1-x^2)}dx \to x = \sin\theta \to dx = \cos\theta\, d\theta$

32. $2\frac{dy}{dx} = \frac{xy+3x-y-3}{xy-2x+4y=8}$

Sol: $\frac{x(y+3)-(y+3)}{x(y-2)+4(y-2)} = \frac{(y+3)\cdot(x-1)}{(y-2)(x+4)} \to \frac{y-2}{y+3}dy = \frac{x-q}{x+4}dx \to \int \frac{y-2}{y+3}dy = \int \frac{x+1}{x+4}dx$
$\to \int \left(1 - \frac{5}{y+3}\right)dy = \int \left(1 - \frac{5}{x+4}\right)dx \to y - 5\ln|y+3| = x - 5\ln|x+4| + C$

Ecuaciones diferenciales homogéneas de 2º orden

- Si son del tipo: $Ay'' + By' + Cy = 0$ con coeficientes constantes

  ***Ejemplo*: $y'' - 5y' + 6y = 0$** cond iniciales: si x=0 → y=3 si x=1 → y=4

  Cambiamos a su ecuac. característica: $x^2 - 5x + 6 = 0$ resolvemos: $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} \to x = 2\,;\ x = 3$

  Las soluciones son números reales sin duplicidad → solución de la ec. característica es la combinación lineal de: $y_1 = e^{2x}$; $y_2 = e^{3x}$ → $\boldsymbol{y = \lambda e^{2x} + \beta e^{3x}}$ $(\lambda, \beta \in \mathbb{R})$

  Sustituimos las cond, iniciales para hallar λ, β: $3 = \lambda e^0 + \beta e^0$; $4 = \lambda e^2 + \beta e^3$

- Si son del tipo: $Ay'' + By' + Cy = 0$ con coeficientes variables → hacer cambio de variable y'=u ; y''=u'

  ***Ejemplo*: $x^4 y'' + 5x^3 y' = 0$**

  cambio de variable y'=u ; y''=u': $x^4 u' + 5x^3 u = 0$ separamos variables: $u'/u = -5x^3/x^4$

  $\int \frac{du}{u} = \int -\frac{5}{x}dx \to \ln u = -5(\ln x + \ln C) \to \cancel{\ln} u = \cancel{\ln}(Kx)^{-5} \to u = (Kx)^{-5}$

  descambiamos: $y' = (Kx)^{-5} \to \frac{dy}{dx} = Kx^{-5} \to \int dy = k\int x^{-5}dx \to y = k(\frac{-1}{4x^4+C})$

- Si son del tipo: $y'' = f(x)$ → hacer cambio de variable y'=u

  ***Ejemplo*: $y'' = -3x^2 + 4x = 0$** condiciones iniciales *y'(0) = 2* e *y(1) = 3*

  cambio de variable y'=u ; y''=u': $u' = -3x^2 + 4x \to \frac{du}{dx} = -3x^2 + 4x \to \int du = \int(-3x^2 + 4x)dx$

  $u = -x^3 + 2x^2 + C$; si *y'(0) = 2 :* $y'(0) = -0 + 2\cdot 0 + C = 2 \to C = 2$

  $y' = -x^3 + 2x^2 + 2 \to \frac{dy}{dx} = -x^3 + 2x^2 + 2 \to \int dy = \int(-x^3 + 2x^2 + 2)dx \to$

  $y = -\frac{x^4}{4} + 2\frac{x^3}{3} + 2x + C \to y(1) = -\frac{1}{4} + \frac{2}{3} + 2 + C = 3 \to C = \frac{7}{12} \to \boldsymbol{y = -\frac{x^4}{4} + 2\frac{x^3}{3} + 2x + \frac{7}{12}}$